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Development of Height-age and Site-index Functions for Even-aged Interior Douglas-fir in British Columbia

by
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ABSTRACT

Height-age and site-index curves were developed from stem analysis of 262 dominant trees in 68 plots located in even-aged Douglas-fir stands in the interior of British Columbia. Study plots were located throughout the geographic range of the species in the Interior Douglas-fir, Interior Cedar-Hemlock, and Sub-Boreal Spruce biogeoclimatic zones. Plots ranged in site index from 7.8 to 30.4 m at 50 years breast-height age, and in age from 64 to 347 years. A polymorphic conditioned logistic function was selected for predicting height as a function of breast-height age and site index; a linear model was selected for predicting site index as a function of height and breast-height age. The height-age and site-index curves were slightly different from polymorphic curves developed from stem analysis data in nearby northern Idaho and greatly different from existing anamorphic curves developed from temporary sample plot data in the interior of British Columbia.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS iii

ABSTRACT iii

INTRODUCTION 1

METHODS 1

- Plot Data 1
- Height-Age Curves 3
- Site-Index Curves 5

RESULTS AND DISCUSSION 5

- Height-Age Curves 5
  - Model selection 5
  - Curve comparisons 9
  - Number of years to reach breast height 12
- Site-Index Curves 13
  - Model selection 13
  - Curve comparisons 17

CONCLUSIONS 18

APPENDIX 1. Plot and stand statistics for the 68 stem analysis plots 19

APPENDIX 2. Table of site height by breast-height age and site index generated with Equation 9 20

APPENDIX 3. Table of site index by height and breast-height age generated with Equation 11 21

LITERATURE CITED 22

TABLES

1 Interior Douglas-fir stem analysis plot and stand statistics by biogeoclimatic zone 2

2 Number of interior Douglas-fir plots by 5-m site-index class and biogeoclimatic zone 4

3 Regression statistics for the height-age models 6

4 Regression statistics for the five site-index models 15

FIGURES

1 Location of the 68 interior Douglas-fir stem analysis plots 2

2 Breast-height age curves for the 68 plots 3

3 Height-age curves from the four models: conditioned logistic, logistic, conditioned Chapman-Richards, and Chapman Richards 6

4 Interior Douglas-fir height-age curves extrapolated to 350 years with the four old-growth plots from the Sub-Boreal Spruce biogeoclimatic zone 7

5 Mean and two standard errors by breast-height age for residuals of observed plot height minus predicted height from the conditioned logistic formulation 8

6 Mean and two standard errors by 5-m site-index class for residuals of observed plot height minus predicted height from the conditioned logistic formulation 8

7 Comparison of the conditioned logistic height-age curves with the Ministry of Forests' (1981) curves 10

8 Comparison of the conditioned logistic height-age curves with Monserud's (1984) curves for wet habitats, mesic habitats, and dry habitats 11

9 Mean and two standard errors by breast-height age for residuals of plot height minus the predicted height from Monserud's (1984) curves for dry habitats 12

10 Relationship between years to reach breast height and site index 13

11 Site-index curves for even-aged interior Douglas-fir 14

12 Mean and two standard errors by breast-height age for residuals of plot site index minus site index predicted with the formulated model by breast-height age 14

13 Mean and two standard errors by 5-m site-index class for residuals of plot site index minus site index predicted with the formulated model 15

14 Site-index functions inverted to portray height as a function of age 16

15 Mean residuals of plot site index minus site index predicted from the conditioned logistic formulation, the Ministry of Forests' model, and Monserud's models for dry habitats, mesic habitats, and wet habitats 17

INTRODUCTION

The ability to predict accurately the growth and yield of future stands and forests is fundamental to the management of timber resources. Site-index and height-age curves are basic components in most growth and yield systems. Site-index curves are used to estimate the productive capacity of the land, and height-age curves are used to predict the pattern of height growth for dominant trees in even-aged stands of a given site index. Many growth and yield models rely on site index to define the height growth patterns of dominant trees which is closely related to volume growth (e.g., Mitchell 1975; Arney 1985; Mitchell and Cameron 1985).

Dominant height and site index of even-aged interior Douglas-fir in British Colombia have been estimated with the site-index curves developed by the B.C. Ministry of Forests (1981). These curves were developed from temporary sample plot data and formulated with an anamorphic function. However, when tested against stem analysis and permanent sample plot data, the anamorphic curves did not reflect actual height growth patterns. As a result, height growth projections and site-index estimates have been seriously biased, especially after index age. This problem is typical of curves developed using temporary sample plot data and anamorphic functions (Spurr 1952; Carmean 1975; Monserud 1985). Because the anamorphic height-age curves were used for predicting height and site index, both estimates were biased and could not be used confidently with height-driven methods of yield prediction.

The objective of our study was to develop new height-age and site-index functions from stem analysis data that reflect the actual growth patterns of dominant trees in even-aged stands of interior Douglas-fir. These functions are intended to provide unbiased estimates of height and site index for use in growth and yield modelling for silvicultural decision-making, timber supply scheduling, and timber supply analysis.

This paper describes the technical aspects of the development of the height-age and site-index functions. The data, analyses, and model development are documented in sufficient detail for other researchers to follow the methods. The application of the curves and functions is emphasized in Thrower and Goudie (1992).

METHODS

Plot Data

Candidate stands in which the height growth of even-aged, interior Douglas-fir trees could be examined through stem analysis were identified from Ministry of Forests (MOF) inventory data. The stands had greater than 80% Douglas-fir by volume and were older than 60 years of age. Field inspection of the selected stands resulted in the location of 68 fixed-area plots (0.04 ha) in 50 well-stocked, undamaged, even-aged stands throughout the geographic range of the species (Figure 1). Thirty-four plots located in the Interior Douglas-fir (IDF) biogeoclimatic zone and 24 plots located in the Interior Cedar-Hemlock (ICH) zone (Pojar *et al.* 1987) were measured in the fall and winter of 1987–88. An additional 10 plots located in the Sub-boreal Spruce (SBS) zone were measured in the fall and winter of 1988–89. Geographic and site data were recorded and plots were classified to the biogeoclimatic subzone level. Trees greater than 7.5 cm diameter at breast height (dbh) were tallied by species and measured for dbh, crown class, width, and length. Four site trees were selected as the tallest, dominant, undamaged trees. The trees were felled, total height measured, and bole sections cut at heights of 0.3 and 1.3 m, and at nine equally spaced intervals above 1.3 m. Disks were labelled and the total number of rings on each disk were counted in the laboratory. Table 1 gives stand and plot statistics for the 68 plots by biogeoclimatic zone. Appendix 1 gives statistics for the individual plots, whose plot numbers are shown in Figure 1.

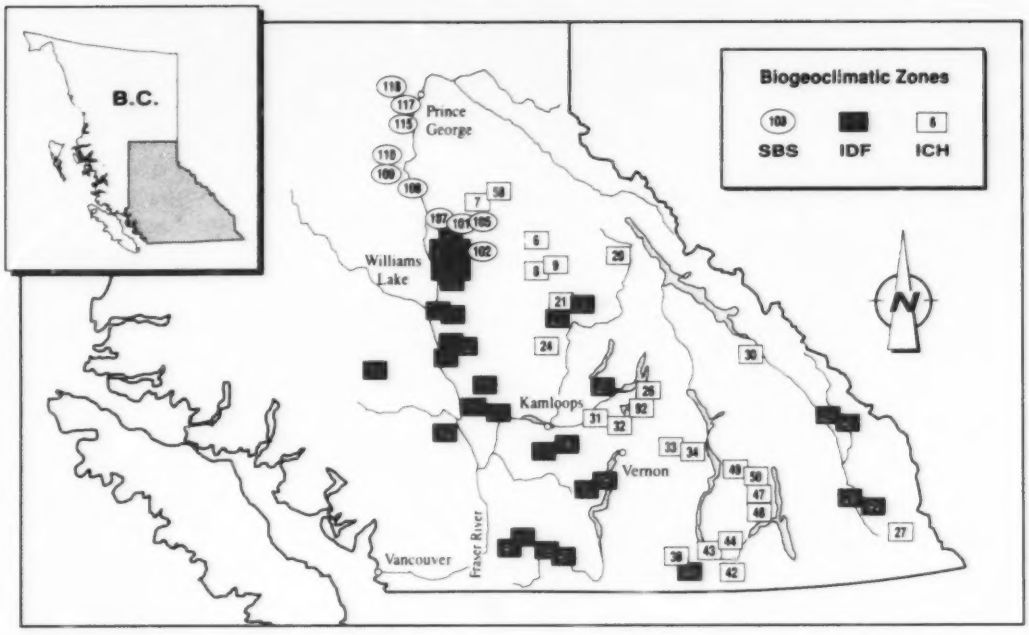

FIGURE 1. Location of the 68 interior Douglas-fir stem analysis plots. Plot numbers are shown by biogeoclimatic zone and referenced in Appendix 1.

TABLE 1. Interior Douglas-fir stem analysis plot and stand statistics by biogeoclimatic zone

	Mean	Min.	Max.	Std. Dev.
IDF zone (34 plots)				
Total age[a] (years)	103.5	68	155	20.9
Stems/ha	798	200	1450	318
Spacing factor[b] (%)	20.6	11.0	67.0	13.0
Quadratic mean dbh (cm)	23.8	15.0	38.8	5.3
Basal area/ha (m^2)	34.5	7.1	56.9	13.9
Fd basal area[c] (% of plot)	95	69	100	8.5
Years to breast height	11.6	5.8	21.3	3.3
Site index (m at 50 years bh age)	15.5	7.8	27.9	4.3
ICH zone (24 plots)				
Total age (years)	91.7	64	111	13.7
Stems/ha	749	175	1300	321
Spacing factor (%)	14.7	9.0	32.0	5.3
Quadratic mean dbh (cm)	28.0	17.4	44.2	6.8
Basal area/ha (m^2)	41.2	20.8	75.4	13.6
Fd basal area (% of plot)	78	52	98	13.6
Years to breast height	8.3	5.8	13.8	1.7
Site index (m at 50 years bh age)	20.9	9.8	30.4	5.4
SBS zone (10 plots)				
Total age (years)	170.1	96	347	85.3
Stems/ha	645	175	1250	359
Spacing factor (%)	13.4	8.0	21.0	3.5
Quadratic mean dbh (cm)	37.8	23.6	64.5	12.4
Basal area/ha (m^2)	58.4	43.0	90.8	15.5
Fd basal area (% of plot)	84	63	100	14.8
Years to breast height	9.4	7.0	11.5	1.5
Site index (m at 50 years bh age)	17.9	13.7	24.7	3.5

[a] Average total age from seed for site trees, assuming 4 years to stump height (0.3 m).
[b] Average spacing between trees (assuming square spacing) as a percentage of dominant height.
[c] Douglas-fir basal area as a percentage of plot basal area.

Height-Age Curves

Height-age curves were constructed for each tree based on total age and breast-height age. The height at which each disk was cut from the tree was adjusted to correct for the bias that occurs when disks are not cut exactly at annual nodes (Carmean 1972). The corrected heights were then plotted against total age and examined for logical errors, abnormal height-age patterns within trees, and differing height-age patterns among trees within plots. We estimated total age by adding four years to the stump age (age of the 0.3 m disk). Ten site trees were suspected of damage or suppression and were deleted from the analyses.

To develop breast-height age curves for each tree, the number of years to reach breast height was subtracted from the total-age curve. An average breast-height age curve was constructed for each plot by averaging the heights of the breast-height age curves for the site trees at 5-year intervals (estimated by linear interpolation between the corrected heights). We then computed plot average height at each 5-year interval, using at least three trees when four site trees were in a plot, at least two trees for the six plots having only three site trees in the plot, and all trees for the two plots having only two trees. This accounts for the very small upward or downward shift in the plot average curve at the last 5-year interval for some plots (Figure 2). Plot site index was defined as the average height of the site trees at 50 years breast-height age. Table 2 gives the number of plots by 5-m site-index class and biogeoclimatic zone.

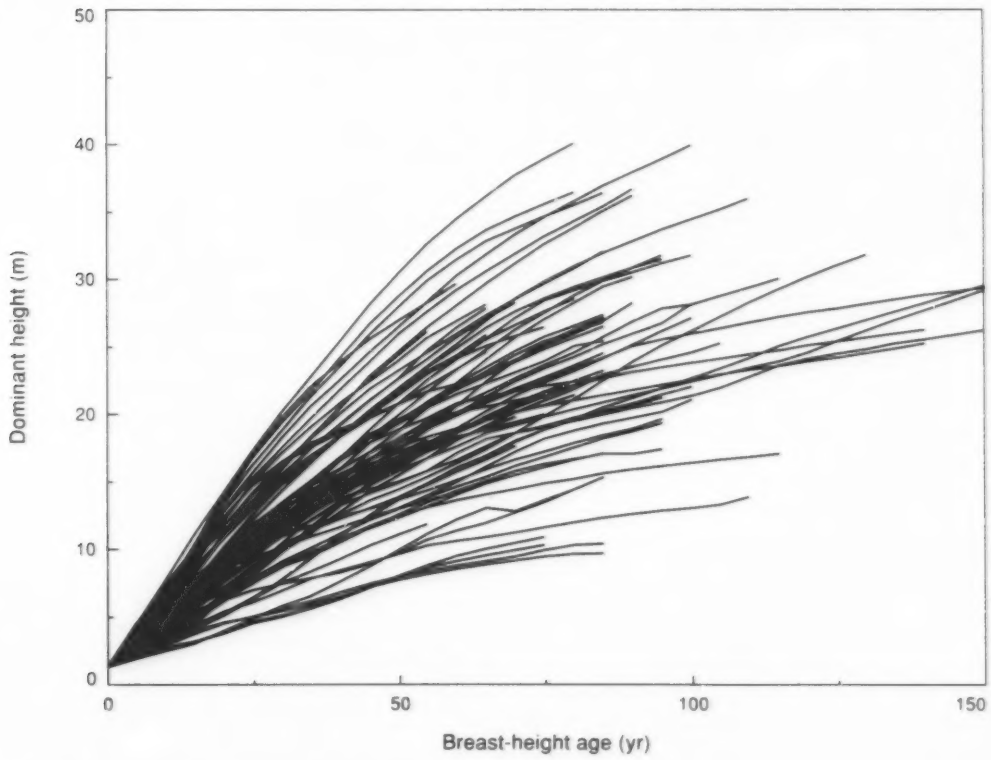

FIGURE 2. Breast-height age curves for the 68 plots.

TABLE 2. Number of interior Douglas-fir plots by 5-m site-index class and biogeoclimatic zone

Biogeoclimatic zone	Site-index class midpoint and range (m)					Total
	10 7.5-12.4	15 12.5-17.4	20 17.5-22.4	25 22.5-27.4	30 27.5-32.4	
IDF	7	17	9		1	34
ICH	2	3	9	8	2	24
SBS		5	4	1		10
Total	**9**	**25**	**22**	**9**	**3**	**68**

Four different nonlinear models were tested for their ability to describe the plot average breast-height age curves. The objective was to select the model that best fit the data over the range of ages and site indices, as well as to select a model that gave realistic extrapolations beyond the range of age. The models were fit with nonlinear, ordinary least squares and a derivative-free algorithm (SAS Institute Inc. 1988). The distribution of the data at older ages was unbalanced and contained only plots of medium site quality. Consequently, only the data up to 100 years breast-height age were included in the analyses to provide a more even distribution of height across the range of site indices and ages (Figure 2).

The four height-age models were:

- a five-parameter Chapman-Richards function (Ek 1971)

$$H = 1.3 + b_1 (S - 1.3)^{b_2} (1 - e^{b_3 A})^{\, b_4 (S - 1.3)^{b_5}} \quad (1)$$

- a conditioned five-parameter Chapman-Richards function (Newnham 1988)

$$H = 1.3 + b_1 (S - 1.3)^{b_2} (1 - k^{A/50})^{\, b_4 (S - 1.3)^{b_5}} \quad (2)$$

where:

$$k = 1 - ((S - 1.3)/b_1 (S - 1.3)^{b_2})^{1/b_4 (S - 1.3)^{b_5}}$$

- a modified logistic function (Monserud 1984)

$$H = 1.3 + b_1 (S - 1.3)^{b_2} / (1 + e^{(b_3 + b_4 \ln A + b_5 \ln(S - 1.3))}) \quad (3)$$

- and a conditioned logistic function[1]

$$H = 1.3 + (S - 1.3) \left[\frac{1 + e^{b_3 + b_4 \ln 50 + b_5 \ln(S - 1.3)}}{1 + e^{b_3 + b_4 \ln A + b_5 \ln(S - 1.3)}} \right] \quad (4)$$

where H is dominant height (m), S is site index (dominant height [m] at 50 years breast-height age), e is the base of the natural logarithm, A is breast-height age (years), ln is the natural logarithm, and b_i are regression coefficients. The conditioned models (Eqs. 2 and 4) are constrained to go through site index at index age.

Primary evaluation of the four height-age models was based on a graphical comparison of residual scatterplots. Plots of residuals over breast-height age, site index, and predicted values by 5-m site-index classes were examined for bias and variance. The models were also compared when extrapolated to 350 years of age. The curves resulting from the model that provided the best fit to the data and gave the most realistic extrapolations were compared graphically to the MOF curves (B.C. Ministry of Forests 1981) and the curves for northern Idaho (Monserud 1984) and Montana and central Washington (Vander Ploeg and Moore 1989). The MOF and Monserud curves were also compared with the stem analysis data through the use of residual scatterplots.

[1] Goudie, J.W. 1984. Height growth and site index curves for lodgepole pine and white spruce and interim managed stand yield tables for lodgepole pine in British Columbia. Final Report FY-1983-84 submitted to the B.C. Min. For., Res. Br., Victoria, B.C. Unpubl. man. 75 pp.

Site-Index Curves

Four nonlinear models and one linear model were tested for predicting site index as a function of height and breast-height age. The data were restricted to those observations between 15 and 100 years of age. Observations at 5 and 10 years of age were not included because site index is usually not strongly related to total height and breast-height age below about 15 years of age.

The four nonlinear models were:

- a modified five-parameter Chapman-Richards function (Payendeh 1974)

$$S = b_1 H^{b_2} (1 - e^{b_3 A})^{b_4 H^{b_5}} \quad (5)$$

- a model used by Carmean and Lenthall (1989)

$$S = b_1 (50 - A) A^{b_2} + b_3 H + b_4 H A^{-1} \quad (6)$$

- a model used by Smith[2]

$$S = H (A/50)^{b_1} + b_2 H^{b_3} (1 - (A/50)^{b_4}) \quad (7)$$

- and a conditioned logistic function

$$S = H \left[\frac{1 + e^{b_3 + b_4 \ln 50 + b_5 \ln H}}{1 + e^{b_3 + b_4 \ln A + b_5 \ln H}} \right] \quad (8)$$

where the variables are previously defined.

We developed a linear site-index model using Dahms' (1975) method to generate a list of candidate predictor variables. Site index was described for each 5-year age class using three different models as a function of height and age:

$$S = a_1 + a_2 H$$

$$S = b_1 + b_2 H + b_3 H^2$$

$$\ln S = c_1 + c_2 \ln H$$

The coefficients from these equations were then formulated as a function of age. The models describing the coefficients were substituted back into the equations and the resulting terms were used as candidate predictor variables in a linear regression of site index as a function of height and breast-height age using the data from all 5-year age classes.

The five site-index models were evaluated on their fit to the data and their ability to extrapolate beyond the range of the data. The site-index curves were inverted to height-age curves using an iterative procedure to determine the height at a given site index and age, and then they were graphically compared to the final height-age curve.

RESULTS AND DISCUSSION

Height-Age Curves

Model selection

We selected the conditioned logistic function (Eq. 4) to describe the dominant height growth of even-aged interior Douglas-fir. The conditioned logistic formulation is:

$$H = 1.3 + (S - 1.3) \left[\frac{1 + e^{5.78009 - 1.15004 \ln 50 - 0.23772 \ln(S - 1.3)}}{1 + e^{5.78009 - 1.15004 \ln A - 0.23772 \ln(S - 1.3)}} \right] \quad (9)$$

where H is dominant height (m), S is site index (dominant height [m] at 50 years breast height age), e is the base of the natural logarithm, ln is the natural logarithm, and A is breast-height age (years). Appendix 2 gives a table of heights generated from Equation 9 for various ages and site indices.

[2] Smith, N.J. 1988. Height growth and site index equations for interior lodgepole pine. Contract to the B.C. Min. For., Res. Br., Victoria, B.C. Unpubl. man., 4 p. + app.

The height growth portrayed by the four models was very similar within the range of the data and gave similar mean squared errors (Table 3). The curves diverge, however, beyond 100 years, especially at high sites (Figure 3). The modified and conditioned logistic functions (Eqs. 3 and 4) gave curves that were very

TABLE 3. Regression statistics for the height-age models

Model	Equation	Mean squared error	Degrees of freedom
Chapman-Richards	1	0.71685	1146
Chapman-Richards–conditioned	2	0.71483	1147
Logistic	3	0.71423	1146
Logistic–conditioned	4	0.71506	1148

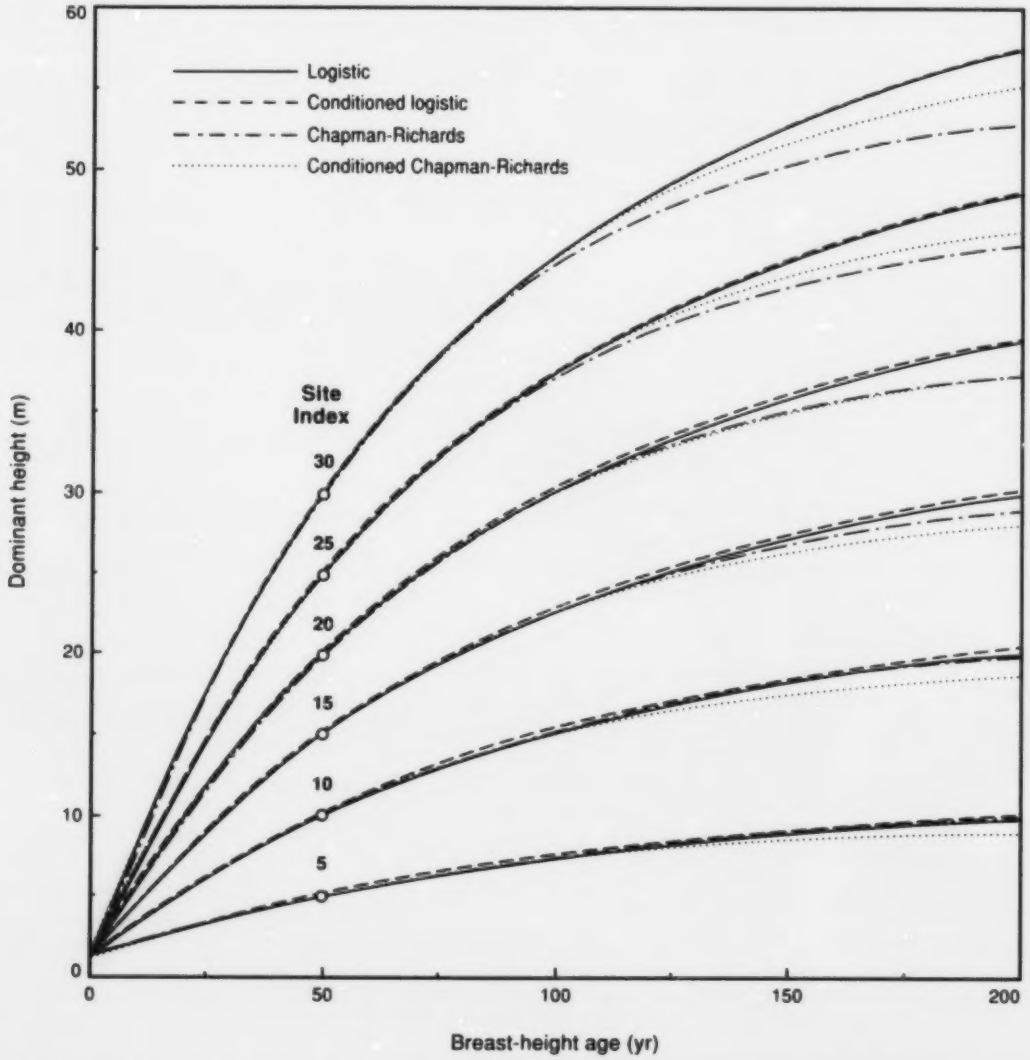

FIGURE 3. Height-age curves from the four models: conditioned logistic (dashed), logistic (solid), conditioned Chapman-Richards (dotted), and Chapman-Richards (dot-dashed). Curves are extrapolated beyond 100 years.

similar even when extrapolated to 350 years, but the conditioned model has the computational advantage of two fewer parameters. The Chapman-Richards functions (Eqs. 1 and 2) showed much lower height growth than the logistic functions for medium and good sites when extrapolated beyond 100 years (Figure 3). The pattern of height growth for the four old-growth plots from the SBS zone suggested that the rate of growth on medium sites was not declining as rapidly as indicated by the Chapman-Richards formulations, and that dominant height growth of older stands might be more realistically portrayed by the logistic formulation (Figure 4).

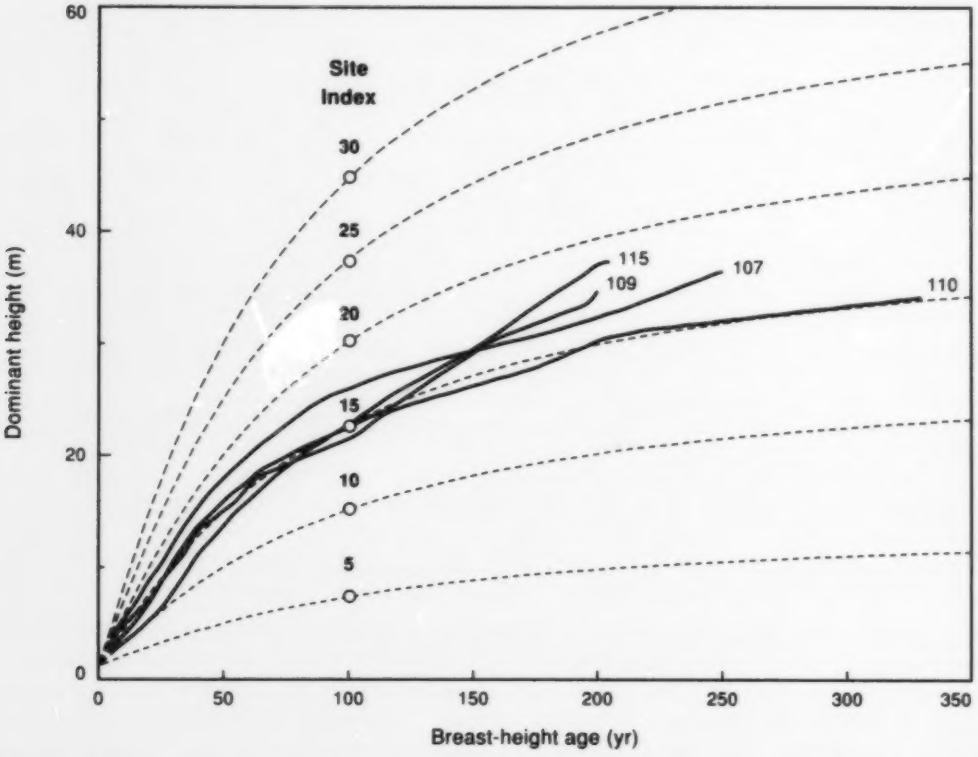

FIGURE 4. Interior Douglas-fir height-age curves (conditioned logistic function) extrapolated to 350 years with the four old-growth plots from the Sub-Boreal Spruce biogeoclimatic zone. Plot numbers are given for each plot curve.

Although the conditioned logistic height-age model (Eq. 9) was generally unbiased over age for the range of the data, both logistic formulations showed a small negative bias at the first 5-year interval of height (Figure 5). This slight under-prediction of height at early ages did not occur with the Chapman-Richards formulations. The conditioned logistic height-age model was generally unbiased over site-index class, but slightly over-predicted height for the 30-m class (Figure 6). Only three plots were in the 30-m site-index class (Table 2) and the large negative mean residual relative to the other site-index classes was caused by a single plot that rapidly decreased in height growth in the last decade. All four models showed a slight over-prediction of height for the 30-m site-index class.

We chose the conditioned logistic over the unconditioned formulation because it gave virtually the same fit to the data with two fewer parameters, not because it is conditioned to predict the site-index height at site-index reference age. Conditioning a height-age function to pass through site-index height at index age has no biological significance. Such conditioning imposes an artificial constraint on the model that usually results in a poorer fit to the data and poorer estimates of site index than when a separate site-index function is developed (Curtis *et al.* 1974).

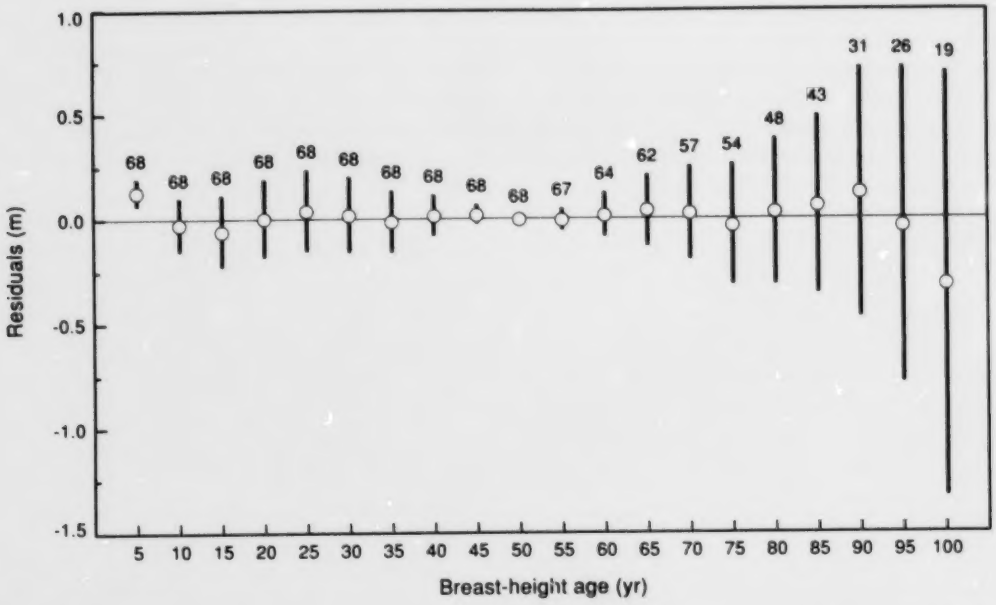

FIGURE 5. Mean and two standard errors by breast-height age for residuals of observed plot height minus predicted height from the conditioned logistic formulation (Eq. 9). The number of observations is given above for each error bar.

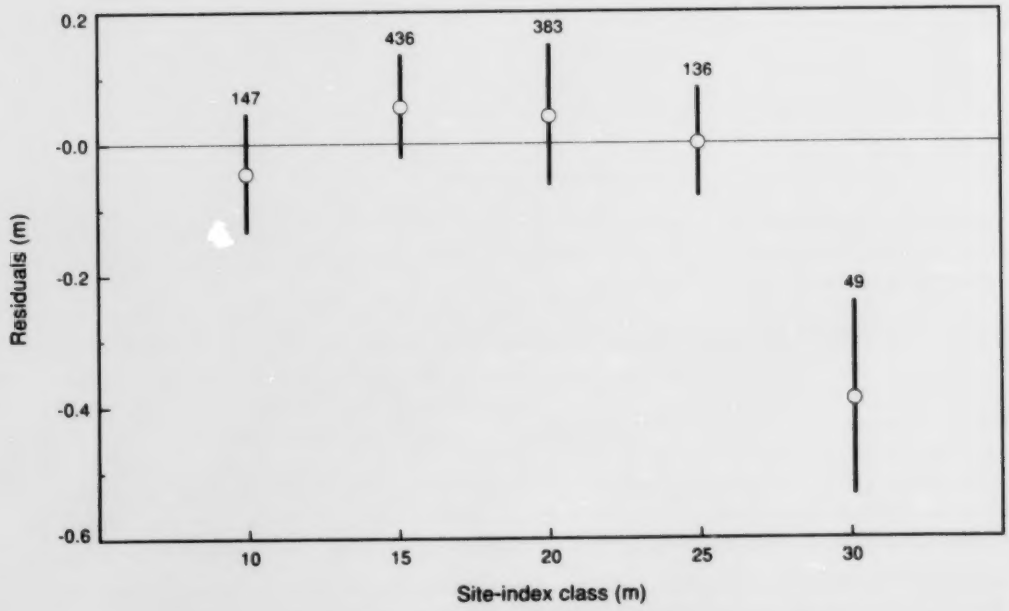

FIGURE 6. Mean and two standard errors by 5-m site-index class for residuals of observed plot height minus predicted height from the conditioned logistic formulation (Eq. 9). The number of observations is given above each error bar.

Monserud (1984) chose an unconditioned logistic function to describe height growth of interior Douglas-fir in northern Idaho. He found that the Chapman-Richards model (Eq. 1) was biased and generally not flexible enough to reflect polymorphism across sites for his data. We did not encounter this problem, however. This difference might be attributed to the different range of ages considered (Monserud's data was up to 200 years breast-height age) or slightly different patterns of height growth.

The height growth portrayed by the conditioned logistic function (Eq. 9) for the 30-m site-index class may be too high when extrapolated beyond the range of the data (Figure 3). This possibly optimistic estimate of height growth on high sites could result in improbably high estimates of growth and yield from height-driven stand simulation models (e.g., Mitchell 1975; Arney 1985; Mitchell and Cameron 1985). This optimistic prediction of height will also result in conservative, under-predictions of site index for older, high site stands when the height-age curves are used for estimating site index.

Curve comparisons

The conditioned logistic height-age curves were different from the B.C. Ministry of Forests' (1981) anamorphic curves (Figure 7), Monserud's (1984) polymorphic curves (Figure 8), and Vander Ploeg and Moore's (1989) polymorphic curves (not shown). The MOF curves showed slightly greater heights before index age (50 years breast-height age) and much lower heights after that. Our height-age curves were very similar to Monserud's curves at site index 30 for wet habitats, at site index 25 for mesic habitats, and at site index 20 for dry habitats. Our curves showed lower heights after index age than did Monserud's curves for site indices less than about 20 m. Vander Ploeg and Moore's (1989) curves showed slightly lower heights than our curves after about 70 years.

The large difference between our curves and the MOF curves was not unexpected. The latter were formulated with an anamorphic Chapman-Richards function from temporary sample plot data. As other studies have shown, height-age curves developed from temporary sample plot data and anamorphic functions typically differ from curves developed through stem analysis or with the use of permanent sample plot data and polymorphic functions (Spurr 1952; Carmean 1975; Monserud 1985). Anamorphic curves usually over-predict height before index age and under-predict height after index age. The corollary is that site index is under-predicted before index age and over-predicted after index age.

We did not, however, expect the difference between our curves and Monserud's curves, because they were both developed using similar methods from geographically contiguous regions. Our data showed lower height growth at older ages on low sites and less polymorphism. The difference is apparent well within the range of our data and is greater in the extrapolated region (beyond 100 years breast-height age). Figure 9 shows the residuals for the plot average height-age curves (based on stem analysis) minus the predicted height from Monserud's (1984) curves for dry habitats. Positive residuals indicate that Monserud's model under-predicted the height of the plot data for a given age and site index. Residual plots determined from Monserud's curves for the mesic and moist habitat types showed even greater bias.

The difference between our curves and Vander Ploeg and Moore's (1989) curves was small within the common range of data. Their data included 14 plots up to 80 years (total age) for Montana and 16 plots up to 87 years (total age) for central Washington. Their curves showed lower height growth when extrapolated to the limit of our data (100 years breast-height age) and beyond. The differences between the curves may be due to the different range of data. Vander Ploeg and Moore found that the height growth of Douglas-fir was similar to that portrayed by Monserud's curves within the same geographic area. For Montana and central Washington, their curves showed lower height growth than did Monserud's curves.

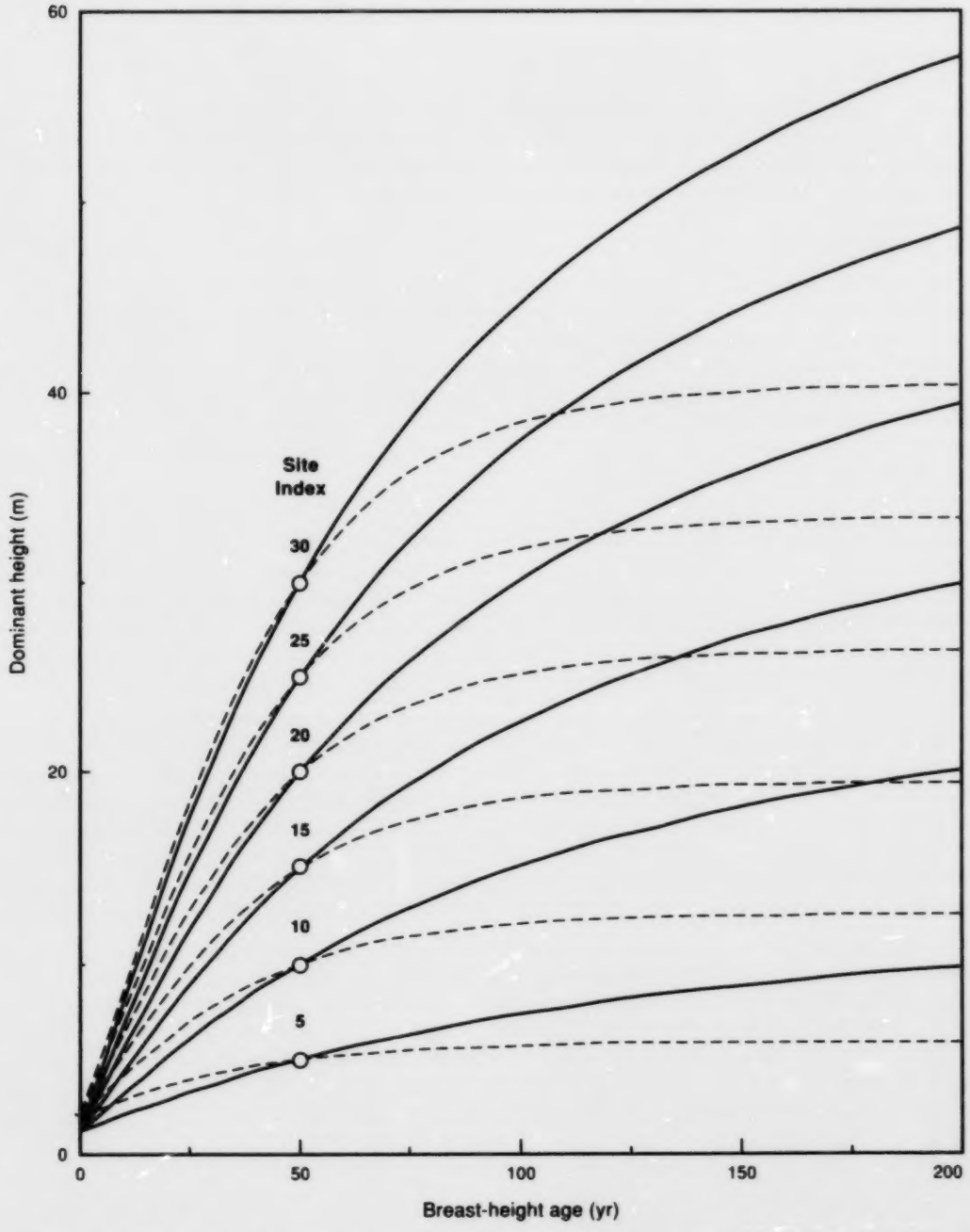

FIGURE 7. Comparison of the conditioned logistic height-age curves (solid) with the Ministry of Forests' (1981) curves (dashed).

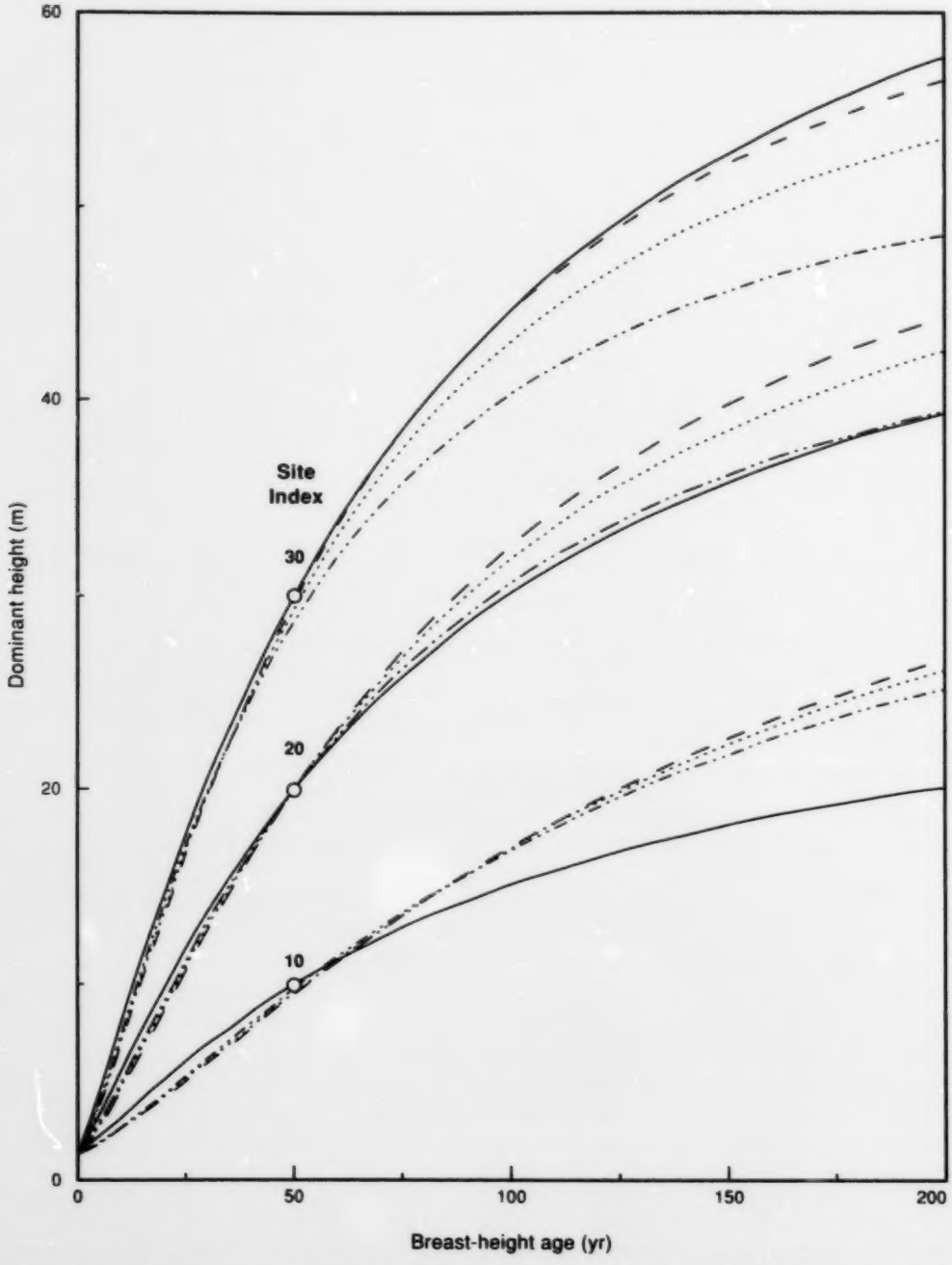

FIGURE 8. Comparison of the conditioned logistic height-age curves (solid) with Monserud's (1984) curves for wet habitats (dashed), mesic habitats (dotted), and dry habitats (dot-dashed).

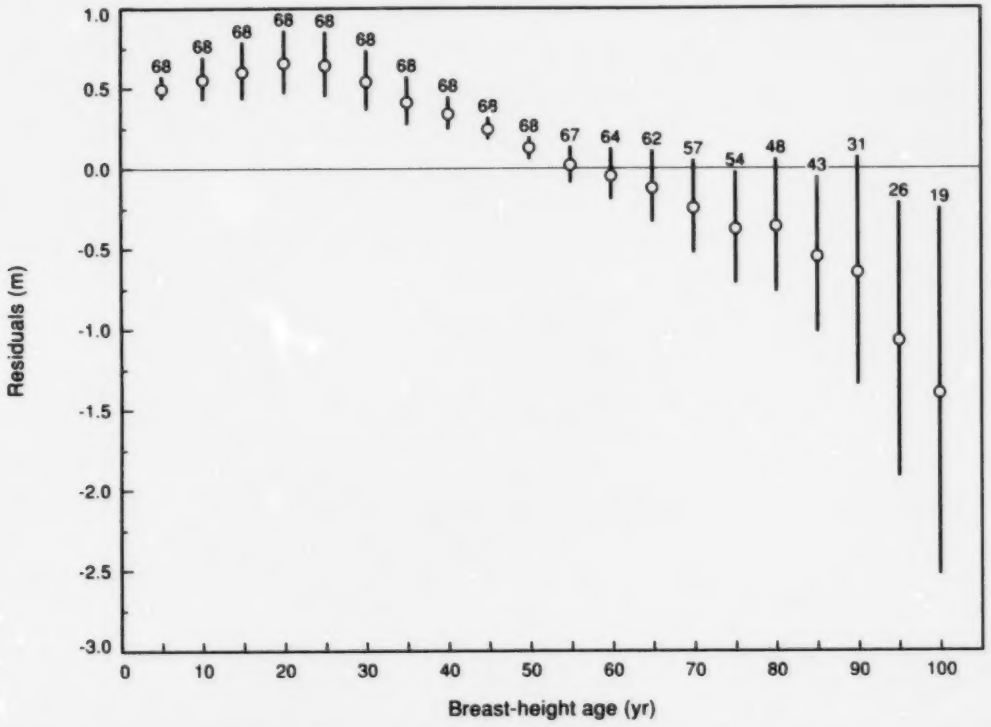

FIGURE 9. Mean and two standard errors by breast-height age for residuals of plot height minus the predicted height from Monserud's (1984) curves for dry habitats. The number of observations is given above each error bar.

Number of years to reach breast height

The average number of years for the site trees in each plot to reach breast height varied from about 6 to 21 years (Figure 10). The average number of years for even-aged Douglas-fir to reach breast height (*yrbh*) can be estimated by:

$$yrbh = yrst + 98.97\ S^{-1} \qquad (10)$$

where *yrst* is the average number of years to reach stump height (0.3 m) and *S* is site index (Figure 10). We assumed an average of 4 years to reach stump height, but this assumption may not be appropriate for some areas. To adjust Equation 10 for local conditions, one can substitute the appropriate estimated number of years to reach stump height.

The apparent strong relationship between years to breast height and plot site index suggests that Douglas-fir trees growing on high sites reach breast height sooner than trees growing on poor sites. The inherent site differences in growing capacity will further support this relationship in the absence of other factors; however, non-site factors can often confound this relationship. Good sites often show a wide variation in the number of years to reach breast height, especially for shade-tolerant species where trees are not killed by severe brush competition. A possible explanation for the lack of Douglas-fir plots on good sites showing a large number of years to reach breast height is that juvenile interior Douglas-fir trees will not withstand severe competition on the wetter, high sites. Thus, trees only survive on good sites where brush competition is low. This apparently strong relationship may also reflect sampling bias.

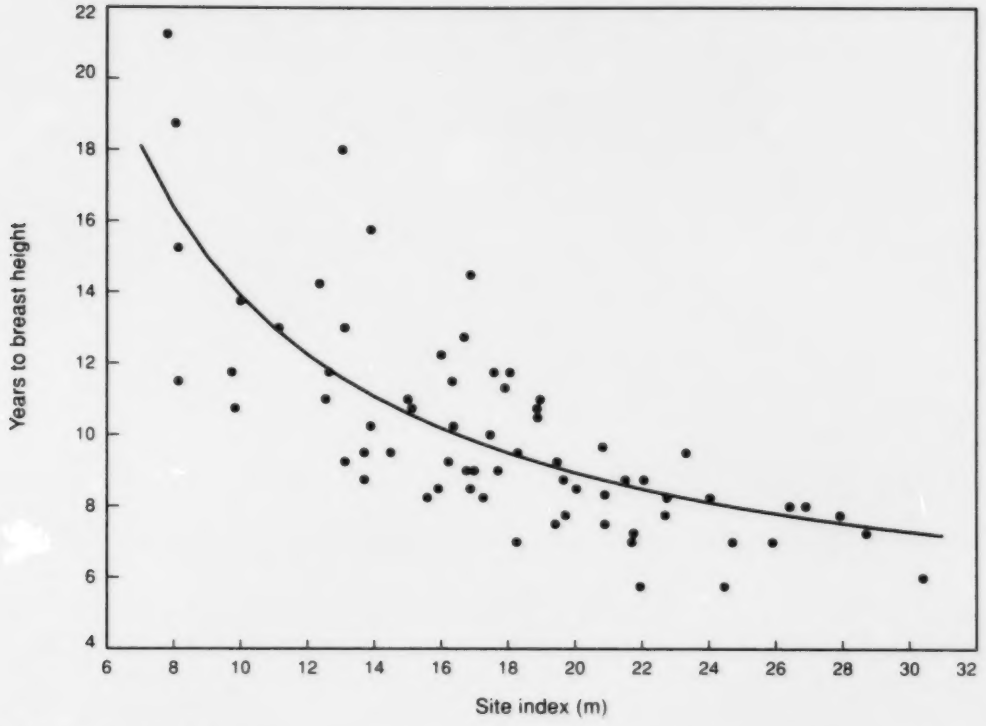

FIGURE 10. Relationship between years to reach breast height and site index. Solid line is the number of years to reach breast height predicted with Equation 10.

Site-Index Curves

Model selection

A linear model developed using predictor variables generated with Dahms' (1975) method was selected for predicting site index. The model is:

$$S = 0.39 + 0.3104\ H + 33.3828\ HA^{-1} \qquad (11)$$

where S is site index (dominant height [m] at 50 years breast-height age), H is dominant height (m), and A is breast-height age (years). Figure 11 shows the site-index curves and Appendix 3 gives a table of site indices by age and height from Equation 11. This model was generally unbiased over age (Figure 12), but showed a small bias over site-index class (Figure 13). Positive residuals indicate under-prediction of the model.

The site-index formulations from the models used by Payendeh (Eq. 5), Carmean and Lenthall (Eq. 6), and Smith (Eq. 7) were about equally as precise as the linear model (Eq. 11) within the range of the data (15–100 years breast-height age) (Table 4). Carmean and Lenthall's model was the most accurate for the few data after 100 years breast-height age. The conditioned logistic gave the most accurate estimates of site index at 5 and 10 years breast-height age, but was very poor at other ages.

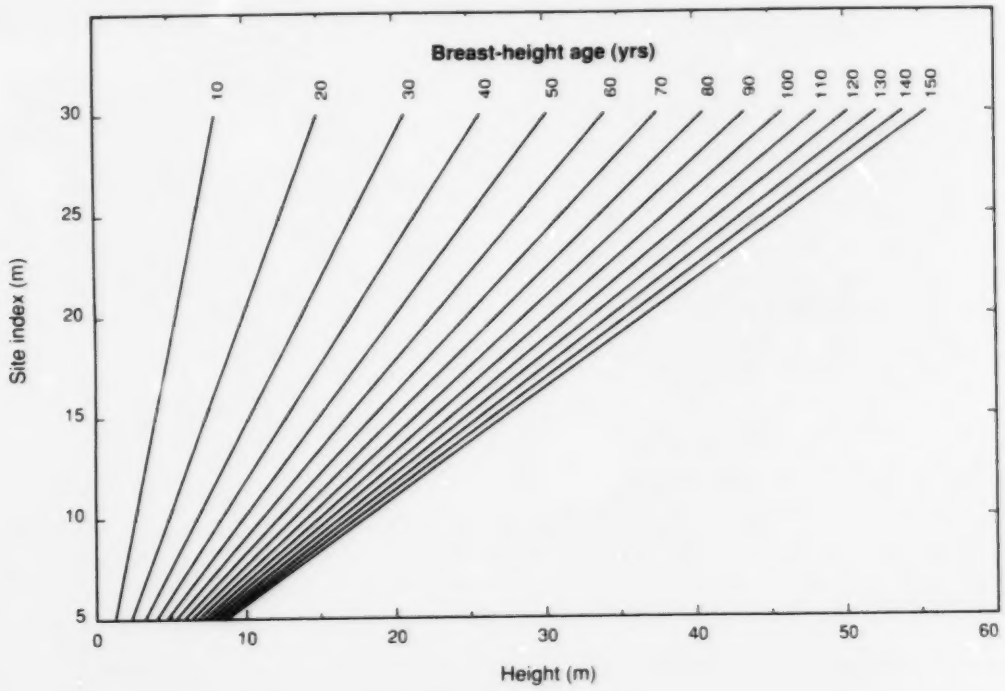

FIGURE 11. Site-index curves for even-aged interior Douglas-fir.

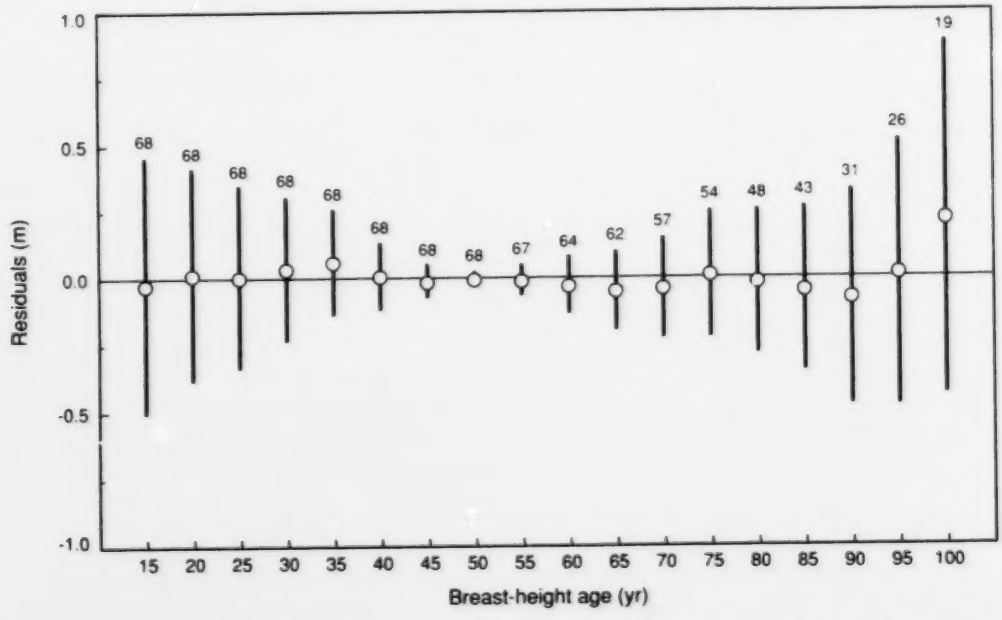

FIGURE 12. Mean and two standard errors by breast-height age for residuals of plot site index minus site index predicted with the formulated model (Eq. 11) by breast-height age. The number of observations is given above each error bar.

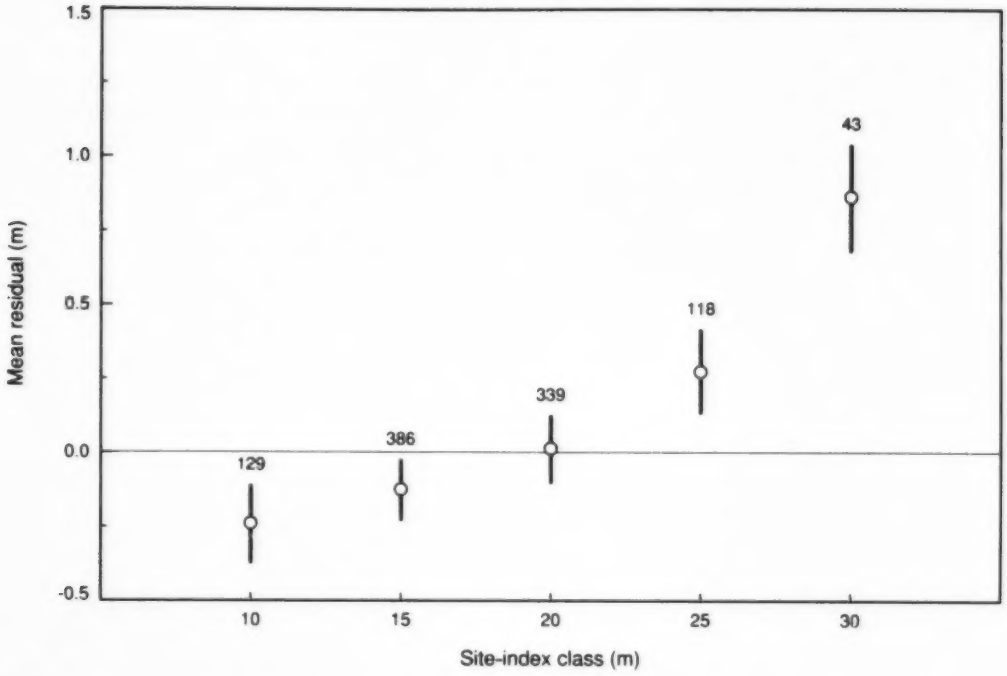

FIGURE 13. Mean and two standard errors by 5-m site-index class for residuals of plot site index minus site index predicted with the formulated model (Eq. 11). The number of observations is given above each error bar.

TABLE 4. Regression statistics for the five site-index models

Model	Equation	Mean squared error	Degrees of freedom
Linear	11	1.024	1037
Payendeh	5	1.029	1039
Carmean and Lenthall	6	1.035	1046
Smith	7	1.035	1047
Conditioned logistic	8	1.161	1175

Figure 14 shows the five site-index models inverted to portray height as a function of site index and age compared to the conditioned logistic height-age curve. All curves showed very similar patterns except the logistic (not shown), which was much higher after index age.

We chose the linear model to predict site index because it was the simplest of the five models and it explained a significant portion of the variation in site index (R^2 = 0.96). The nonlinear models explained slightly more variation than the linear model, but the differences were very small within the range of the data. Like Monserud (1984), we found that Dahms' (1975) method gave linear models that were severely over-parameterised. However, the method provided a list of transformations of height and age from which we could select predictor variables. Monserud found that the Chapman-Richards site-index model (Eq. 5) was seriously biased for his site-index data. As for the height-age model, we did not have this problem with the Chapman-Richards function and our data.

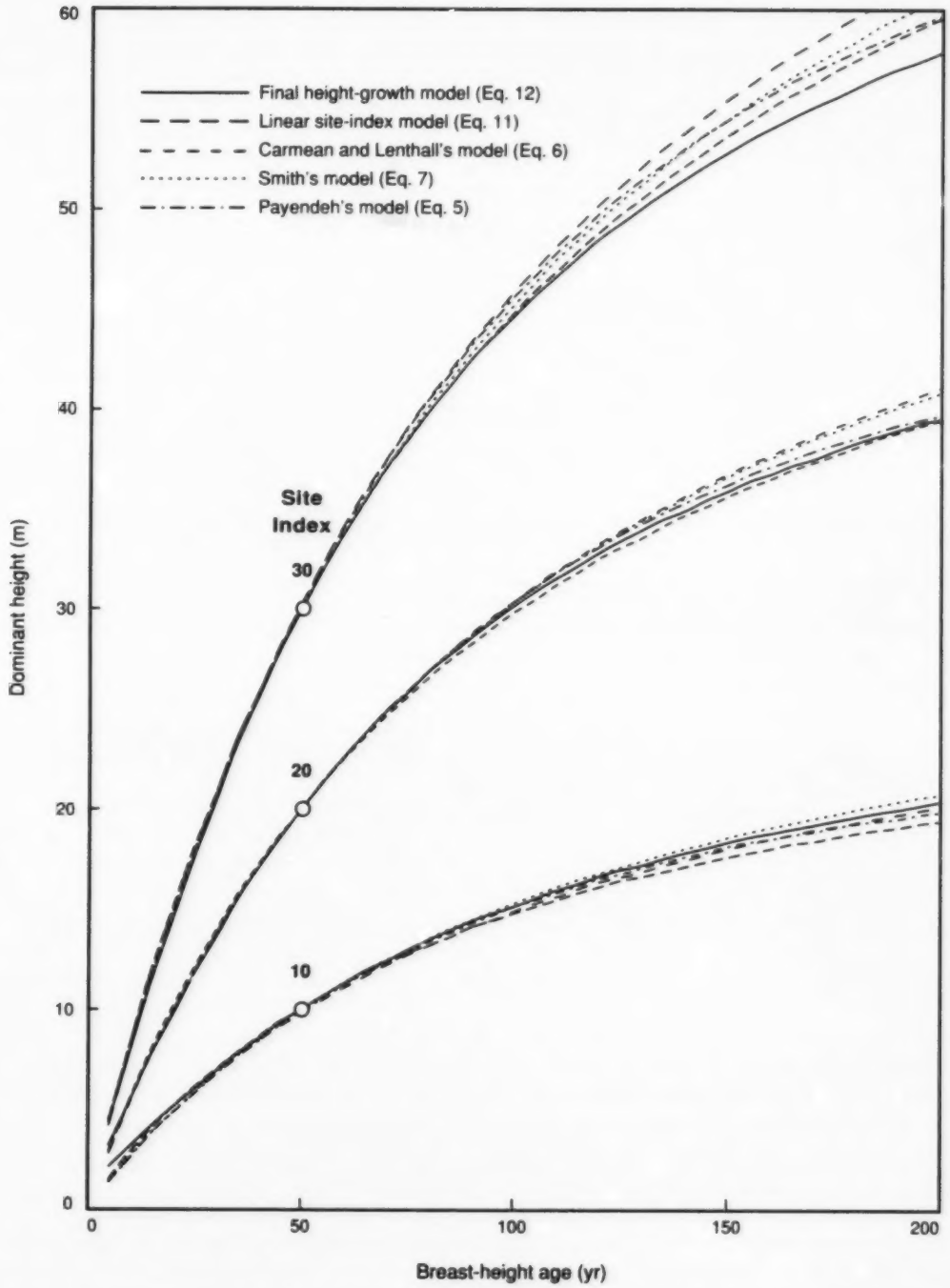

FIGURE 14. Site-index functions inverted to portray height as a function of age. Conditioned logistic (Eq. 9 - solid), linear site-index model (Eq. 11 - long dash), Smith's model (Eq. 7 - dotted), Payendeh's model (Eq. 5 - dot-dashed), and Carmean and Lenthall's model (Eq. 6 - short dash).

Curve comparisons

Estimates of site index from our site-index curves differed notably from estimates of site index from the MOF height-age curves and Monserud's site-index curves. Figure 15 shows the mean site index residuals computed as the plot site index minus the site index predicted with the various curves for a given height and breast-height age. Positive residuals indicate under-prediction of the various models. The MOF height-age curves over-predicted height before index age and under-predicted height after index age. Site index for the plots was therefore under-predicted before index age and over-predicted after index age. Monserud's height-age curves generally showed lower height growth before index age and higher height growth after index age than did our curves, and thus plot site index was over-predicted before index age and under-predicted after index age.

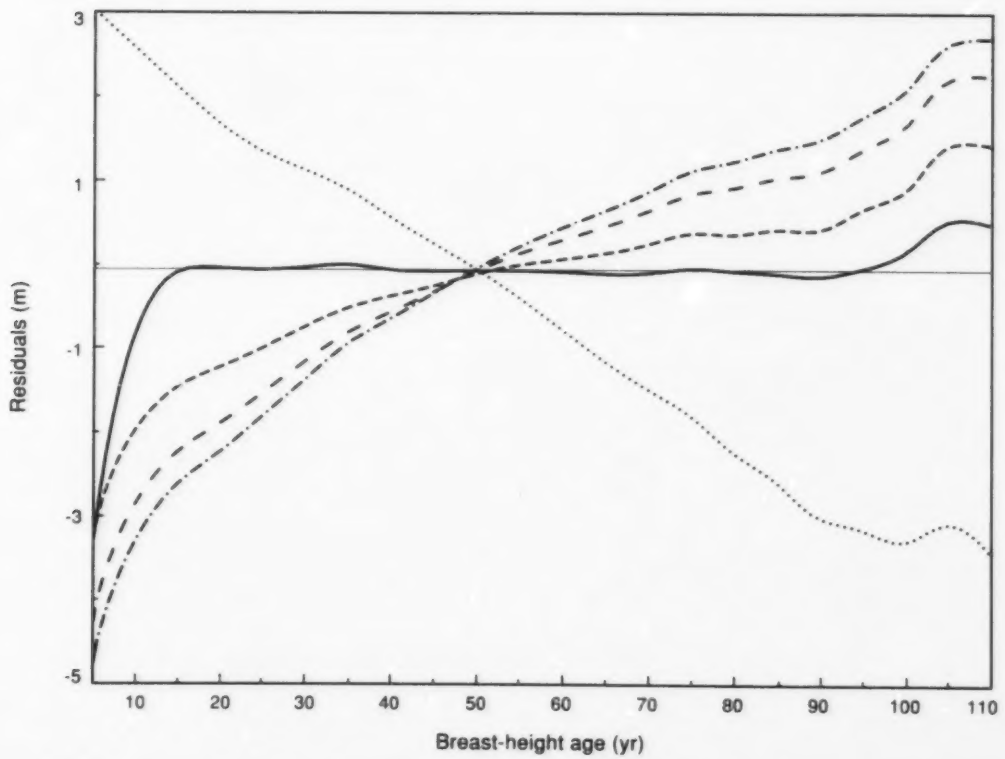

FIGURE 15. Mean residuals of plot site index minus site index predicted from the conditioned logistic formulation (solid), the Ministry of Forests' model (dotted), and Monserud's models for dry habitats (short-dashed), mesic habitats (long-dashed), and wet habitats (dot-dashed).

CONCLUSIONS

The height-age and site-index functions developed in this study reflect the height growth patterns of dominant, free-growing trees in the interior of British Columbia and should provide more accurate estimates than did the previously used curves. The curves formerly used in British Columbia (B.C. Ministry of Forests 1981) gave biased estimates of height and site index because they were developed from temporary sample plot data and formulated with an anamorphic function. With the former curves, height was slightly over-predicted below index age (50 years breast-height age) and was dramatically under-predicted after index age. Because those height-age curves were also used for estimating site index, site index was slightly under-predicted below index age and over-predicted after index age.

The slight differences between our curves and Monserud's (1984) may reflect real differences in height growth patterns between British Columbia and neighboring northern Idaho. However, Monserud's curves were also developed using similar techniques of stem-analysis data and formulated with a polymorphic logistic model. This supports Monserud's conclusions that height growth patterns of interior Douglas-fir differ among ecologically different areas in Idaho. This suggests that height growth patterns may also differ among the ecologically diverse area within British Columbia.

Further study into the height growth of interior Douglas-fir in British Columbia should therefore examine the hypothesis that height growth patterns for a given site index vary among ecologically different areas in the province. As well, further sampling should include more plots from high sites to determine whether or not these curves over-predict height growth on high sites.

APPENDIX 1: Plot and stand statistics for the 68 stem analysis plots

Plot no.	BEC zone	Avg. age (yr)	Avg. ht. (m)	Site index (m)	Years to BH[a]	Site trees	BA[b] Fd (%)	Avg. dbhq[c] (cm)	BA per ha (m^2)	Trees per ha	WSF[d] (%)
1	IDF	153	26.45	18.95	9.0	4	100	25.42	50.75	1000	12
2	IDF	131	17.46	12.54	9.0	4	100	24.17	30.96	675	22
3	IDF	110	22.10	13.88	8.3	4	100	24.55	42.81	900	15
4	IDF	85	20.49	16.85	6.5	4	82	20.10	32.54	1025	15
5	IDF	122	11.48	8.05	16.8	4	100	17.95	15.19	600	36
6	ICH	74	28.50	22.77	6.2	4	62	25.40	48.15	950	11
7	ICH	91	24.88	18.26	5.0	4	95	29.82	32.73	475	18
8	ICH	108	40.25	26.43	6.0	4	90	39.30	66.73	550	11
9	ICH	80	29.12	22.69	5.7	4	77	29.01	31.40	475	16
10	IDF	88	21.40	16.77	7.0	4	88	21.61	47.67	1300	13
11	IDF	112	20.80	13.11	11.0	4	100	25.79	36.56	700	18
12	IDF	107	18.05	12.63	9.8	4	100	23.85	33.50	750	20
13	IDF	114	25.37	16.98	7.0	4	87	22.73	51.72	1275	11
14	IDF	87	21.92	18.30	7.5	4	89	19.15	41.78	1450	12
15	IDF	80	23.73	20.84	7.7	3	99	24.23	42.65	925	14
16	IDF	109	27.68	17.46	8.0	4	96	27.99	56.90	925	12
17	IDF	102	23.70	16.20	7.2	4	100	24.40	30.39	650	17
18	IDF	105	29.02	17.58	9.8	4	100	30.65	47.97	650	14
19	IDF	97	28.02	19.45	7.2	4	100	32.75	48.45	575	15
20	ICH	84	27.05	21.68	5.0	4	82	25.10	37.11	750	13
21	ICH	107	31.90	20.88	6.3	3	79	34.63	75.35	800	11
22	IDF	92	36.58	27.93	5.7	4	96	38.82	50.31	425	13
23	IDF	66	18.83	17.26	6.2	4	100	20.01	44.80	1425	14
24	ICH	105	32.53	20.03	6.5	4	57	29.69	67.48	975	10
25	IDF	102	31.90	21.93	3.7	4	87	29.77	48.71	700	12
26	ICH	109	20.65	13.89	6.7	4	87	21.51	42.88	1175	14
27	ICH	97	23.27	15.59	6.2	4	98	40.86	22.95	175	32
28	IDF	84	14.93	13.05	16.0	4	100	17.15	16.75	725	25
29	IDF	91	10.88	8.13	9.5	4	100	17.90	8.17	325	51
30	ICH	62	27.02	24.47	3.7	4	61	29.01	42.98	650	15
31	ICH	96	33.10	23.34	7.5	4	68	23.93	37.12	825	11
32	ICH	95	36.63	25.91	5.0	4	86	26.30	51.62	950	9
33	ICH	72	31.33	26.90	6.0	3	53	22.05	31.50	825	11
34	ICH	73	28.40	24.04	6.2	4	71	23.32	38.44	900	12
35	IDF	81	17.70	13.90	13.8	4	75	20.82	26.39	775	20
36	IDF	68	17.45	16.89	12.5	4	92	17.48	31.80	1325	16
38	IDF	85	21.75	16.69	10.8	4	80	20.20	39.27	1225	13
39	ICH	90	29.53	21.75	5.2	4	69	31.42	44.59	575	14
42	ICH	103	31.47	18.88	8.5	4	72	30.96	33.88	450	15
43	ICH	90	37.25	28.70	5.2	4	86	35.18	29.16	300	15
44	ICH	90	41.37	30.41	4.0	3	87	44.20	38.36	250	15
47	ICH	95	16.04	9.83	8.8	4	52	17.43	29.84	1250	18
48	ICH	91	15.33	10.00	11.8	4	88	21.93	20.78	550	28
49	ICH	71	25.60	21.50	6.7	4	87	22.39	51.20	1300	11
50	ICH	71	24.80	20.89	5.5	4	82	24.69	41.88	875	14
51	IDF	97	27.27	18.87	8.8	4	100	30.20	53.73	750	13
52	IDF	111	18.51	12.36	12.3	4	100	19.15	23.03	800	19
53	IDF	121	10.68	7.82	19.3	4	100	27.55	17.89	300	54
54	IDF	110	22.15	15.11	8.8	4	100	27.73	25.68	425	22
55	IDF	150	25.45	17.68	7.0	4	100	26.34	43.60	800	14
56	IDF	126	13.81	9.73	9.8	4	100	18.60	15.62	575	30
57	IDF	84	20.81	16.37	8.3	4	69	20.05	25.25	800	17
58	ICH	92	26.32	19.41	5.5	4	92	24.46	30.55	650	15
90	IDF	123	30.37	18.07	9.8	4	97	30.73	38.95	525	14
91	IDF	100	23.90	16.02	10.3	4	100	23.93	30.37	675	16
92	ICH	107	19.95	13.11	7.2	4	91	20.19	41.61	1300	14
93	IDF	87	10.63	8.13	13.3	4	100	21.25	7.09	200	67
94	IDF	72	12.83	11.13	11.0	3	81	15.01	17.25	975	25
101	SBS	113	27.80	16.33	9.5	2	70	29.01	47.94	725	13
102	SBS	94	27.05	19.71	5.7	4	88	24.87	44.94	925	12
105	SBS	98	28.05	19.65	6.7	4	100	23.58	50.23	1150	11
107	SBS	261	36.63	17.90	9.3	3	100	64.47	57.12	175	21
108	SBS	95	36.05	24.71	5.0	4	63	35.25	53.68	550	12
109	SBS	206	33.80	15.91	6.5	4	65	41.03	42.98	325	16
110	SBS	345	34.30	15.02	9.0	2	87	43.69	63.71	425	14
115	SBS	212	37.55	13.89	7.5	4	95	46.97	90.98	525	12
117	SBS	117	36.05	22.05	6.7	4	72	28.39	79.11	1250	8
118	SBS	140	32.30	14.48	7.5	4	97	41.30	53.58	400	15

[a] BH - breast height (1.3 m).
[b] BA - basal area.
[c] dbhq - quadratic mean diameter at breast height.
[d] WSF - Wilson's spacing factor (average spacing as a percentage of top height).

APPENDIX 2: Table of site height (m) by breast-height age and site index generated with Equation 9

Breast-height age	Site index (m)										
	10	12	14	16	18	20	22	24	26	28	30
0	1.30	1.30	1.30	1.30	1.30	1.30	1.30	1.30	1.30	1.30	1.30
5	2.17	2.39	2.61	2.83	3.06	3.28	3.51	3.74	3.98	4.21	4.45
10	3.17	3.62	4.09	4.56	5.03	5.51	5.99	6.47	6.96	7.45	7.95
15	4.16	4.85	5.56	6.26	6.98	7.70	8.42	9.15	9.89	10.62	11.37
20	5.12	6.04	6.97	7.91	8.85	9.80	10.75	11.71	12.68	13.64	14.61
25	6.05	7.18	8.32	9.47	10.63	11.79	12.95	14.13	15.30	16.48	17.66
30	6.93	8.26	9.60	10.95	12.30	13.66	15.02	16.38	17.75	19.13	20.50
35	7.76	9.28	10.80	12.33	13.87	15.41	16.95	18.49	20.04	21.59	23.15
40	8.55	10.24	11.94	13.64	15.34	17.04	18.75	20.46	22.17	23.89	25.60
45	9.30	11.15	13.00	14.86	16.71	18.57	20.43	22.29	24.15	26.02	27.88
50	10.00	12.00	14.00	16.00	18.00	20.00	22.00	24.00	26.00	28.00	30.00
55	10.66	12.80	14.94	17.07	19.21	21.34	23.46	25.59	27.72	29.84	31.97
60	11.29	13.56	15.82	18.08	20.33	22.59	24.83	27.08	29.32	31.56	33.80
65	11.88	14.27	16.65	19.03	21.39	23.76	26.11	28.47	30.82	33.17	35.51
70	12.44	14.94	17.43	19.91	22.39	24.85	27.31	29.77	32.22	34.66	37.11
75	12.97	15.58	18.17	20.75	23.32	25.88	28.44	30.99	33.53	36.07	38.60
80	13.47	16.17	18.86	21.54	24.20	26.85	29.50	32.13	34.76	37.38	39.99
85	13.94	16.74	19.52	22.28	25.03	27.76	30.49	33.20	35.91	38.61	41.30
90	14.39	17.28	20.14	22.98	25.81	28.62	31.42	34.21	36.99	39.77	42.53
95	14.82	17.78	20.72	23.64	26.54	29.43	32.30	35.16	38.01	40.85	43.68
100	15.22	18.26	21.28	24.27	27.24	30.19	33.13	36.06	38.97	41.88	44.77
105	15.61	18.72	21.80	24.86	27.90	30.92	33.92	36.91	39.88	42.84	45.80
110	15.97	19.15	22.30	25.42	28.52	31.60	34.66	37.71	40.74	43.76	46.76
115	16.32	19.57	22.78	25.96	29.11	32.25	35.37	38.47	41.55	44.62	47.68
120	16.65	19.96	23.23	26.46	29.68	32.86	36.03	39.18	42.32	45.44	48.54
125	16.97	20.33	23.66	26.95	30.21	33.45	36.67	39.86	43.05	46.21	49.36
130	17.27	20.69	24.06	27.40	30.72	34.00	37.27	40.51	43.74	46.95	50.14
135	17.56	21.03	24.45	27.84	31.20	34.53	37.84	41.13	44.39	47.64	50.88
140	17.84	21.35	24.82	28.26	31.66	35.03	38.38	41.71	45.02	48.31	51.58
145	18.10	21.66	25.18	28.66	32.10	35.51	38.90	42.27	45.61	48.94	52.25
150	18.35	21.96	25.52	29.04	32.52	35.97	39.40	42.80	46.18	49.54	52.89
155	18.59	22.24	25.84	29.40	32.92	36.41	39.87	43.31	46.72	50.12	53.49
160	18.83	22.52	26.15	29.75	33.30	36.83	40.32	43.79	47.24	50.67	54.07
165	19.05	22.78	26.45	30.08	33.67	37.23	40.76	44.26	47.73	51.19	54.63
170	19.26	23.03	26.74	30.40	34.02	37.61	41.17	44.70	48.21	51.69	55.15
175	19.47	23.27	27.01	30.71	34.36	37.98	41.56	45.12	48.66	52.17	55.66
180	19.67	23.50	27.28	31.00	34.68	38.33	41.94	45.53	49.09	52.63	56.15
185	19.86	23.72	27.53	31.28	34.99	38.67	42.31	45.92	49.51	53.07	56.61
190	20.04	23.94	27.77	31.55	35.29	38.99	42.66	46.30	49.91	53.49	57.06
195	20.22	24.14	28.01	31.81	35.58	39.30	43.00	46.66	50.29	53.90	57.48
200	20.39	24.34	28.23	32.07	35.85	39.60	43.32	47.00	50.66	54.29	57.90

APPENDIX 3: Table of site index (m) by height and breast-height age generated with Equation 11

Breast-height age	Site Height (m)																				
	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46
10	22	30	37	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
15	16	21	26	31	36	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
20	12	16	20	24	28	32	36	40	-	-	-	-	-	-	-	-	-	-	-	-	-
25	10	14	17	20	23	27	30	33	37	40	-	-	-	-	-	-	-	-	-	-	-
30	9	12	15	17	20	23	26	29	32	35	37	40	-	-	-	-	-	-	-	-	-
35	8	11	13	16	18	21	23	26	28	31	33	36	38	-	-	-	-	-	-	-	-
40	7	10	12	14	16	19	21	23	26	28	30	32	35	37	39	-	-	-	-	-	-
45	7	9	11	13	15	17	19	21	24	26	28	30	32	34	36	38	40	-	-	-	-
50	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	-	-	-
55	6	8	10	11	13	15	17	19	21	22	24	26	28	30	32	33	35	37	39	-	-
60	6	7	9	11	13	14	16	18	19	21	23	25	26	28	30	32	33	35	37	39	40
65	5	7	9	10	12	14	15	17	19	20	22	23	25	27	28	30	32	33	35	37	38
70	5	7	8	10	11	13	15	16	18	19	21	22	24	26	27	29	30	32	33	35	37
75	5	6	8	9	11	12	14	16	17	19	20	22	23	25	26	28	29	31	32	34	35
80	5	6	8	9	11	12	13	15	16	18	19	21	22	24	25	27	28	29	31	32	34
85	5	6	7	9	10	12	13	14	16	17	19	20	21	23	24	26	27	29	30	31	33
90	-	6	7	9	10	11	13	14	15	17	18	19	21	22	24	25	26	28	29	30	32
95	-	6	7	8	10	11	12	14	15	16	18	19	20	22	23	24	26	27	28	30	31
100	-	6	7	8	9	11	12	13	15	16	17	18	20	21	22	24	25	26	27	29	30
105	-	5	7	8	9	10	12	13	14	15	17	18	19	20	22	23	24	26	27	28	29
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200	-	-	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	19	20	21	22

LITERATURE CITED

Arney, J.D. 1985. A modeling strategy for the growth projection of managed stands. Can. J. For. Res. 15:511-518.

B.C. Ministry of Forests. 1981. Site index equations and curves for the major tree species in British Columbia. Inv. Br., Victoria, B.C. For. Inv. Rep. No. 1. 9 p. + app.

Carmean, W.H. 1972. Site index curves for upland oaks in the Central States. For. Sci. 18:102-120.

_____ . 1975. Forest site quality evaluation in the United States. Adv. Agron. 27:209-269.

Carmean, W.H. and D.J. Lenthall. 1989. Height-growth and site-index curves for north central Ontario. Can. J. For. Res. 19:215-224.

Curtis, R.O., D.J. DeMars, and F.R. Herman. 1974. Which dependent variable in site index-height-age regressions? For. Sci. 20:74-87.

Dahms, W.G. 1975. Gross yield of central Oregon lodgepole pine. *In* Management of lodgepole pine ecosystems. Proc. Symp. D.M. Braumgartner (editor) Wash. State Univ., Coop. Exten. Serv., Pullman, Wash., pp. 208-232.

Ek, A.R. 1971. A formula for white spruce site index curves. Univ. Wisc., Madison Wisc. For. Res. Note 161. 2 p.

Mitchell, K.J. 1975. Dynamics and simulated yield of Douglas-fir. For. Sci. Monogr. 17. 39 p.

Mitchell, K.J. and I.R. Cameron. 1985. Managed stand yield tables for coastal Douglas-fir: initial density and precommercial thinning. B.C. Min. For., Res. Br., Victoria, B.C. Land Manage. Rep. No. 31. 69 p. + suppl.

Monserud, R.A. 1984. Height growth and site index curves for inland Douglas-fir based on stem analysis data and forest habitat type. For. Sci. 30:943-965.

_____ . Comparison of Douglas-fir site index and height growth curves in the Pacific Northwest. Can. J. For. Res. 15:673-769.

Newnham, R.M. 1988. A modification of the Ek-Payendeh nonlinear regression model for site index curves. Can. J. For. Res. 18:115-120.

Payendeh, B. 1974. Nonlinear site index equations for several major Canadian timber species. For. Chron. 50:194-196.

Pojar, J., K. Klinka, and D.V. Meidinger. 1987. Biogeoclimatic ecosystem classification in British Columbia. For. Ecol. Manage. 22:119-154.

SAS Institute Inc. 1988. SAS/STAT user's guide. Release 6.03 edition. Cary, N.C., 1028 p.

Spurr, S.H. 1952. Forest inventory. Ronald Press Co., New York, N.Y. 476 p.

Thrower, J.S. and J.W. Goudie. 1992. Estimating dominant height and site index of even-aged interior Douglas-fir in British Columbia. W.J. Appl. For. 7(1):20-25.

Vander Ploeg, J.L.and J.A. Moore. 1989. Comparison and development of height growth and site index curves for Douglas-fir in the Inland Northwest. W. J. Appl. For. 4(3):85-88.

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